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ABSTRACT

The machine developed by E. R. Fitzgerald to investigate the dynamic mechanical properties of solids is reanalyzed to take account of effects due to a shorted turn on the drive tube. The shorted turn arises from an extension of the aluminum portion of the drive tube into the radial magnetic field. The new analysis predicts the anomalous calibration data of the machine as well as the asymmetry and negative values of loss compliance associated with resonant phenomena.

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SUMMARY

An analysis of the Fitzgerald machine is presented which takes cognizance of the shorted turn on the drive tube of the machine. The resulting equations show the effect of shorted turn parameters upon what Fitzgerald refers to as sample properties. The shorted turn is shown to produce the apparent negative damping of the free drive tube calibration data as well as the negative values of the loss compliance data near large resonances.

INTRODUCTION

Fitzgerald has described an apparatus (refs. 1 and 2) which he first used to measure the dynamic mechanical properties of rubber-like solids. The samples to be tested are mounted in compression between the surfaces of a metal drive tube and a large mass called the floating mass as shown in figure 1. Both the drive tube and the floating mass are each suspended from the magnet housing by wires. The wires supporting the drive tube and floating mass allow each of them to move freely along their axis. An alternating current flowing through two sets of coils wound on portions of the drive tube which are located in a radial magnetic field produce an alternating force on the drive tube. This force produces an alternating shearing strain in the two samples. By measuring the drive tube velocity resulting from this alternating force, Fitzgerald (refs. 1 and 2) is able to calculate a dynamic shear modulus or a dynamic compliance for the samples as a function of frequency.

The dynamic compliance data which Fitzgerald reports for several mixtures of polyvinyl chloride and dimethythianthrene appear to be close to what one might expect for a soft rubber-like material. If the samples are somewhat harder, however, the dynamic compliance shows departures from normal. Fitzgerald first reported (ref. 3) dynamic compliance measurements using metallic samples in 1957. The surprising results reported by Fitzgerald were the numerous resonances present in the dynamic

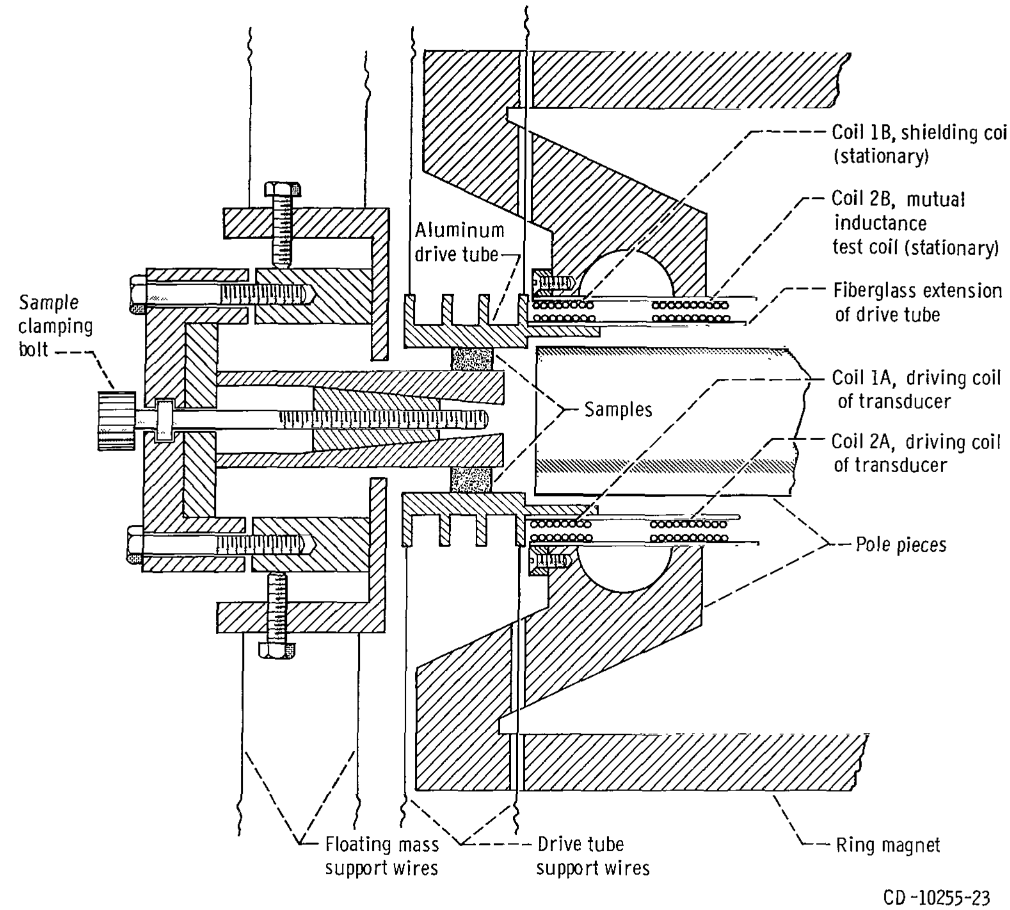


Figure 1. - Simplified view of Fitzgerald apparatus. Note portion of aluminum drive tube that extends into magnetic field.

compliance data for the three metals tested. In subsequent papers, similar resonances are reported by Fitzgerald for polytetrafluorethylene (ref. 4), crystalline polymers (ref. 5), crystalline and fused quartz (ref. 6), ionic crystals (ref. 7), and vinyl and ethyl stearates (ref. 8). In addition to the resonances, the compliance data also exhibits other departures from normal behavior.

The dynamic compliance data of Fitzgerald is represented by two components: a storage compliance J' which is a measure of the energy stored in the system per cycle; and a loss compliance J'' which is a measure of the energy lost in the system per cycle. If the observed resonances are produced by a simple resonator, the loss compliance should always be positive and symmetric about the resonant frequency. The loss compliance reported by Fitzgerald (refs. 3 and 6) for most of the materials he tested is not only unsymmetric about the resonant frequency but it becomes negative on the high frequency side of the resonance for some high frequency resonances (near 3000 Hz). Fitzgerald first points this out in the results for ionic crystals (ref. 7).

A third departure of Fitzgerald's results is the large magnitude of the storage compliance measured at low frequencies where it is expected that close agreement with elastic values should exist. The measured values for most materials (refs. 3, 6, 7, and 9) are 20 to 100 times larger than the accepted values. This is stressed in a summary (ref. 9) of much of his previous results.

Measurements of the dynamic compliance of several ionic crystals were made at this laboratory by Gotsky and Stearns (ref. 10) using an apparatus which was built by Fitzgerald. The results obtained by Gotsky and Stearns exhibited similar resonance effects including the negative values of the loss compliance for some of the resonances. Their results for the low frequency storage compliance were also about 20 to 100 times larger than the accepted values based on elastic constants.

An analysis was undertaken by the present authors in an effort to determine the origin of some of the unusual results reported by the Fitzgerald techniques. During the course of this investigation it was discovered that the drive tube was constructed such that a part of the metal section of the drive tube was located in the radial magnetic field. The metal portion located in the magnetic field constitutes what is called a "shorted turn." Fitzgerald's analysis of his machine does not include the shorted turn effects.

The purpose of this report is to analyze the Fitzgerald machine in the presence of a drive tube with a shorted turn. Equations are derived which show the effect of the shorted turn on calibration and sample data; expressions are developed which relate the true sample properties to the apparent sample properties reported by Fitzgerald. The effect of the shorted turn on resonances in the compliance data is shown to produce the asymmetric and negative loss compliance reported by Fitzgerald.

ANALYSIS

The drive tube of the Fitzgerald apparatus consists of a metal and fiber glass tube (see fig. 1). Two coils of wire, separated by several inches, are wound on the tube. The drive coil, 1A, which normally produces most of the sinusoidal driving force on the drive tube, is wound directly on the aluminum part of the tube; the pick-up coil, 2A, which conducts a smaller alternating current, is wound on a fiber glass extension of the tube. As shown in figure 2, coils 1A and 2A are connected in series with the current limiting resistors R_A and R_3, respectively; each combination is connected to the source of alternating voltage E.

The shorted turn on the drive tube has two sources of induced currents; one is dependent upon drive tube velocity, the other is dependent upon the magnitude of current in coil 1A. As the drive tube moves in response to the alternating current flowing in coils 1A and 2A eddy currents are induced in the shorted turn due to its motion in the

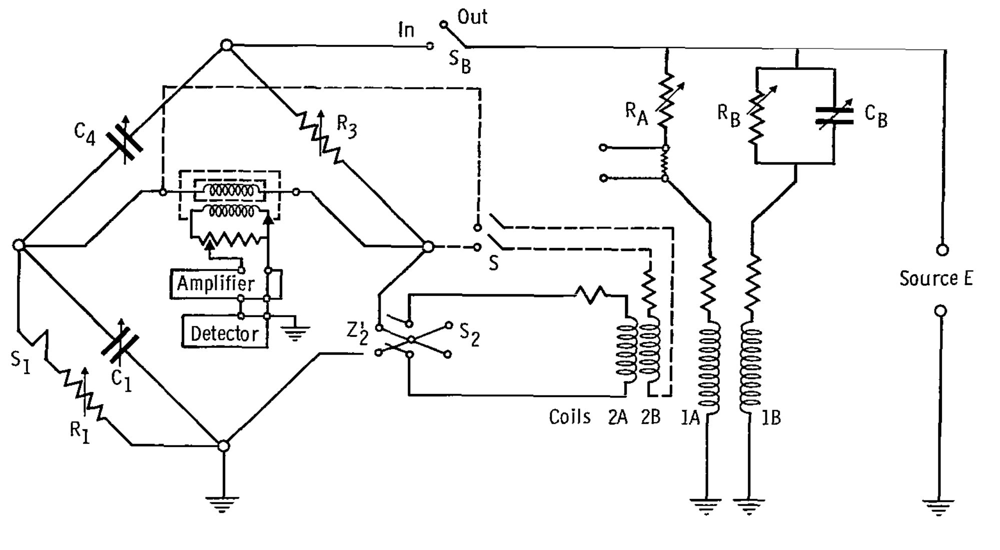


Figure 2. - Electrical circuit connection used in the Fitzgerald apparatus, where coils 1A and 2A are driving coils of transducer, coil 1B is shielding coil (stationary), and coil 2B is mutual inductance test coil.

radial magnetic field. These eddy currents in turn interact with the radial magnetic field to modify the force on the drive tube. The second source of induced currents in the shorted turn is the drive coil 1A itself. Because the drive coil 1A is wound directly on the shorted turn, the shorted turn becomes a single turn secondary of a transformer in which coil 1A is the primary. When an alternating current flows in coil 1A a current is induced in the shorted turn through its mutual inductance with coil 1A. This current also modifies the force on the drive tube.

Coils 1A and 2A are connected in a bridge circuit as shown in figure 2. The dynamic impedance of coil 2A is defined as the ratio of the voltage across coil 2A to the current through coil 2A; the voltage across coil 2A contains an ohmic contribution and a voltage induced by motion of the coil in the radial magnetic field. One method of eliminating the ohmic contribution is to measure the dynamic impedance of coil 2A twice; once with its current flowing in such a manner that the force it produces on the drive tube is in the same direction as that produced by coil 1A, and a second measurement made with the connections of 2A reversed in the bridge circuit. The values of various quantities for the two ways of connecting coil 2A to the bridge circuit are indicated by plus and minus superscripts, where a plus sign indicates the forces produced by the current in coil 2A is in the same direction as the force produced by the current in coil 1A.

The voltages, E_{2A}^{+} and E_{2A}^{-}, developed across coil 2A are composed of the voltage drop due to the current through the coil and the voltage induced by its motion in the magnetic field, or

$$E_{2A}^{\pm} = I_2^{\pm} Z_{2A} \mp D_2 v^{\pm}, \tag{1}$$

where $I_2^{\pm}$ is the current in coil 2A, Z_{2A} is the electrical impedance of coil 2A, $v^{\pm}$ is the velocity of the tube, D_2 is a constant defined below, and velocities, currents, voltages, and impedances are complex quantities.

The voltage induced in either coil and the shorted turn, due to drive tube motion is

$$E_a = \int (\vec{v} \times \vec{B}) \cdot \vec{dl} = \vec{v} \cdot \int \vec{B} \times \vec{dl} \tag{2}$$

where $\vec{v}$ is the velocity of coil element $\vec{dl}$, and $\vec{B}$ is the magnetic induction. The integration is over the total length of the wire of the coil. The quantities D_1, D_2, and D_c are defined by

$$\left.\begin{aligned} D_1 &= \vec{e}_v \cdot \int B_1 \times \vec{dl}_1 \\ D_2 &= \vec{e}_v \cdot \int B_2 \times \vec{dl}_2 \\ D_c &= \vec{e}_v \cdot \int B_c \times \vec{dl}_c \end{aligned}\right\} \tag{3}$$

where $\vec{e}_v$ is a unit vector along $\vec{v}$ and $\vec{B}_1$, $\vec{B}_2$, and $\vec{B}_c$ are respectively the magnetic induction at coils 1A and 2A and the shorted turn.

The current which flows in the shorted turn, $I_c^{\pm}$, is

$$I_c^{\pm} = \frac{1}{Z_c} (-D_c v^{\pm} + Z_{1AC} I_1) \tag{4}$$

where I_1 is the current in coil 1A, Z_c is the impedance of the shorted turn, and Z_{1AC} is an impedance given by

$$Z_{1AC} = -i\omega M_{1AC} \tag{5}$$

where M_{1AC} is the mutual inductance between coil 1A and the shorted turn. Noting that E is much larger than the voltages across the coils, the total force on the drive tube $F^{\pm}$ is

$$F^{\pm} = D_1 I_1 \pm D_2 I_2 + D_c I_c^{\pm} \tag{6}$$

where $I_2^{\pm}$ is approximated by I_2. A mechanical impedance Z_m can be defined as

$$Z_m = \frac{F^+}{v^+} = \frac{F^-}{v^-} \tag{7}$$

Combining equations (4), (6), and (7), $I_c^{\pm}$ is obtained as:

$$I_c^{\pm} = \frac{-\frac{1}{Z_c}\left[\left(\frac{D_1^2}{nZ_m} - Z_{IAC}\right)I_1 \pm \frac{D_1 D_2}{nZ_m} I_2\right]}{1 + \frac{D_1^2}{n^2 Z_m Z_c}} \tag{8}$$

In equation (8) D_c has been approximated by D_1/n where n is the number of turns on coil 1A. Combining equations (6) and (8) yields

$$F^{\pm} = \frac{\left[\left(1 + \frac{Z_{1AC}}{nZ_c}\right) D_1 I_1 \pm D_2 I_2\right]}{1 + \frac{D_1^2}{n^2 Z_m Z_c}} \tag{9}$$

Again using the fact that E is much larger than the voltages across either coil, the coil currents can be approximated by

$$I_1 = \frac{E}{R_A}, \quad I_2 = \frac{E}{R_3} \tag{10}$$

The bridge circuit of figure 2 is used to measure $Z_2^{\pm}$, the dynamic electrical impedance of coil 2A. From equations (1), (7), and (9),

$$Z_2^{+} = \frac{E_{2A}^{+}}{I_2} = Z_{2A} - \frac{\frac{D_2}{Z_m}\left[\left(1 + \frac{Z_{1AC}}{nZ_c}\right) D_1 \frac{I_1}{I_2} + D_2\right]}{1 + \frac{D_1^2}{n^2 Z_m Z_c}} \tag{11}$$

when coil 2A is connected to aid coil 1A and

$$Z_2^- = \frac{E_{2A}^-}{I_2} = Z_{2A} + \frac{\frac{D_2}{Z_m}\left[\left(1 + \frac{Z_{1AC}}{nZ_c}\right)D_1 \frac{I_1}{I_2} - D_2\right]}{1 + \frac{D_1^2}{n^2 Z_m Z_c}} \tag{12}$$

when coil 2A is reversed in the bridge circuit. Subtracting equation (11) from equation (12) and defining ΔZ as this difference, the result is obtained as

$$\Delta Z = Z_2^+ - Z_2^- = \frac{2R_3 D_1 D_2}{R_A Z_m}\left[\frac{\left(1 + \frac{Z_{1AC}}{nz_c}\right)}{1 + \frac{D_1^2}{n^2 Z_m Z_c}}\right] \tag{13}$$

where I_1/I_2 has been replaced by R_3/R_A from equation (10). Equation (13) can be solved for the mechanical impedance Z_m as

$$Z_m = \frac{2R_3 D_1 D_2}{R_A\ \Delta Z}\left(1 + \frac{Z_{1AC}}{nZ_c}\right) - \frac{D_1^2}{n^2 Z_c} \tag{14}$$

By applying equation (14), the mechanical impedance of a system containing a drive tube with a shorted turn could be determined; the usual two bridge measurements, needed to evaluate ΔZ, and knowledge of the shorted turn parameters are required. Equation (14) compares exactly with Fitzgerald's result (refs. 1 and 2) if all shorted turn effects are removed from equation (14) by setting the impedance of the shorted turn Z_c equal to infinity. Thus

$$\underset{Z_c \to \infty}{Z_m} = \frac{2R_3 D_1 D_2}{R_A\ \Delta Z} \tag{15}$$

which is the same as Fitzgerald's result when the product $D_1 D_2$ is replaced by K^2.

DISCUSSION AND APPLICATIONS

The dynamic compliance data of both Fitzgerald (refs. 1-9), and Gotsky and Stearns (ref. 10), were produced on seperate machines each of which had a shorted turn on the drive tube. Unfortunately their analysis did not take into account the shorted turn. Thus all effects of the shorted turn on drive tube velocity are interpreted as sample properties.

In this report Z_{mf} designates a calculated mechanical impedance which is a mixture of sample properties and shorted turn effects. This type of mechanical impedance is obtained by means of Fitzgerald's analysis, in which the shorted turn effects that are present in the measured data are ignored. In order to get a relation for Z_{mf} which expresses the true mechanical impedance Z_m and the shorted turn effects explicitly, equation (13) is used to calculate values of ΔZ in terms of Z_m and the shorted turn parameters. These values of ΔZ are substituted into equation (15) with the result that:

$$Z_{mf} = \frac{Z_m}{1 + \dfrac{Z_{1AC}}{nZ_c}} + \frac{D_1^2}{n^2 Z_c + nZ_{1AC}} \tag{16}$$

where the subscript f indicates that the mechanical impedance, Z_{mf}, was determined from a measured ΔZ which contains shorted turn effects, and an equation which does not take these shorted turn effects into account.

Equation (16) shows the relation between what Fitzgerald calls a mechanical impedance Z_{mf}, the true mechanical impedance Z_m, and the shorted turn parameters. This equation can be used to examine the effects of the shorted turn on the apparent mechanical impedance of the free tube.

The impedance Z_c of the shorted turn can be represented as

$$Z_c = R_c + i\omega L_c \tag{17}$$

where R_c is the resistance of the shorted turn and L_c is its self-inductance. Using equation (5) for Z_{1AC}, equation (16) can be rewritten in the form

$$Z_{mf} = Z_m\left(1 + \frac{i\omega L_c}{R_c}\right) + \frac{D_1^2}{n^2 R_c} \tag{18}$$

where the mutual inductance M_{1AC} between coil 1A and the shorted turn has been approximated as

$$M_{1AC} = nL_c \tag{19}$$

This equation is only correct if all the magnetic flux that threads coil 1A also threads the shorted turn. This of course is not true; but because coil 1A is wound directly on the shorted turn, almost all the flux linking one links the other, and equation (19) should closely approximate the actual case.

The mechanical impedance of the free or unclamped drive tube without a shorted turn can be defined as

$$Z_m^o = \rho^o + i\left(\omega M^o - \frac{S^o}{\omega}\right) \tag{20}$$

where ρ^o is the damping due to air fraction, losses in the support wire, and magnetic damping; M^o is the tube mass, and S^o is the restoring force per unit axial displacement of the tube. By substituting equation (20) into equation (18), the apparent mechanical impedance Z_{mf}^o for the free tube with a shorted turn is

$$Z_{mf}^o = \left(\rho^o + \frac{D_1^2}{n^2R_c} - \frac{\omega^2M^oL_c}{R_c} + \frac{S^oL_c}{R_c}\right) + i\left(\frac{\rho^o\omega L_c}{R_c} + \omega M^o - \frac{S^o}{\omega}\right) \tag{21}$$

As part of the necessary calibration, Fitzgerald defines the two quantities G_{12}^o and B_{12}^o from the mechanical impedance of the free tube as the real and imaginary parts of Z_m^o/K^2, that is

$$G_{12}^o = \frac{\rho^o}{K^2}$$

$$B_{12}^o = \frac{\omega M^o - \dfrac{S^o}{\omega}}{K^2} \tag{22}$$

For a machine with a shorted turn drive tube

$$G^o_{12f} = \frac{1}{K^2}\left(\rho^o + \frac{D_1^2}{n^2 R_c} + \frac{S^o L_c}{R_c} - \frac{\omega^2 M^o L_c}{R_c}\right)$$

$$B^o_{12f} = \frac{1}{K^2}\left(\frac{\rho^o \omega L_c}{R_c} + \omega M^o - \frac{S^o}{\omega}\right) \tag{23}$$

The result to note is the difference between the frequency dependences of G^o_{12} and G^o_{12f}. Equation (22) indicates G^o_{12} should be positive and independent of frequency; however, the model for a shorted turn drive tube predicts G^o_{12f} to be positive at low frequencies and becoming negative at larger frequencies. In figure 3 the real component of Z^o_{mf} is

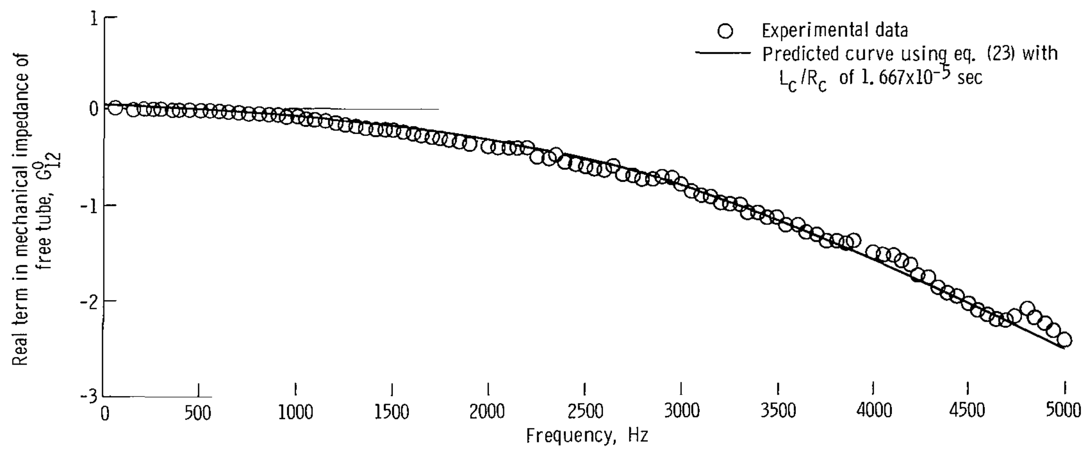


Figure 3. - Comparison of experimental calibration data of real term in mechanical impedance of free tube with that calculated using shorted drive tube analysis.

plotted against frequency, from data obtained on the Fitzgerald apparatus located at this laboratory. The experimental data, represented by open circles, become negative at large frequencies as predicted by the shorted turn analysis; the measured data agrees rather closely with the results calculated by equation (23) and represented in figure 3 by the solid curve. The effect on B^o_{12} produced by the shorted turn is small since $\rho^o L_c/R_c \ll M^o$.

It is of interest now to calculate what effect the shorted turn has on the measured sample mechanical impedance. A quantity Z'_m which Fitzgerald (ref. 1) defines as the mechanical impedance of the sample (and assumes to contain no shorted turn effects) can be obtained by making three measurements with the machine: (1) with the drive tube

unclamped the mechanical impedance Z_m^o is measured, (2) with the drive tube clamped to the floating mass the impedance Z_m^c is measured, and (3) with a sample clamped between the drive tube and the floating mass the impedance Z_m is measured. The sample mechanical impedance Z'_m can then be obtained from Fitzgerald's model of his machine (refs. 1 and 2).

$$Z'_m = \frac{1}{\dfrac{1}{Z_m - Z_m^o} - \dfrac{1}{Z_m^c - Z_m^o}} \tag{24}$$

For a machine with a shorted drive tube, all of the mechanical impedances on the right-hand side of equation (24) will contain shorted turn effects as indicated by equation (18). Using equation (18) in equation (24) an apparent sample mechanical impedance Z'_{mf} can be obtained as

$$Z'_{mf} = Z'_m\left(1 + \frac{i\omega L_c}{R_c}\right) \tag{25}$$

where Z'_m represents the true mechanical impedance of the sample. The damping terms D_1^2/n^2R_c of equation (18) cancel in the derivation of equation (25).

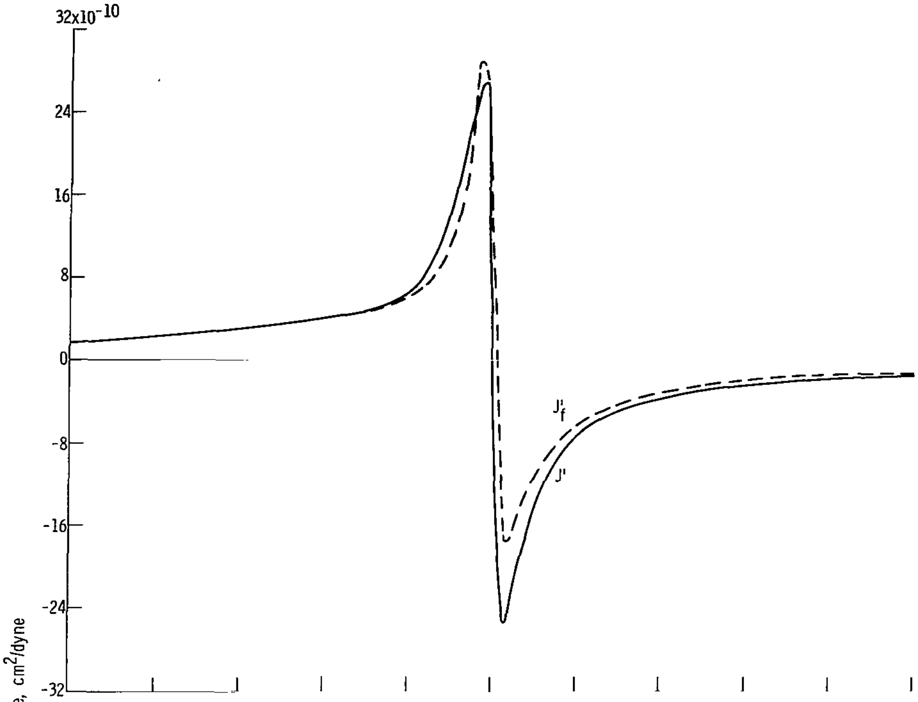
Consider now the effect of the shorted turn drive tube upon the resonances exhibited in the compliance data of Fitzgerald. In considering the resonances observed by Fitzgerald, it is convenient to consider one of them as resulting from an independent oscillator; the mechanical impedance, Z_m^R of such an oscillator can be represented by:

$$Z_m^R = \rho + i\left(\omega m - \frac{k}{\omega}\right) \tag{26}$$

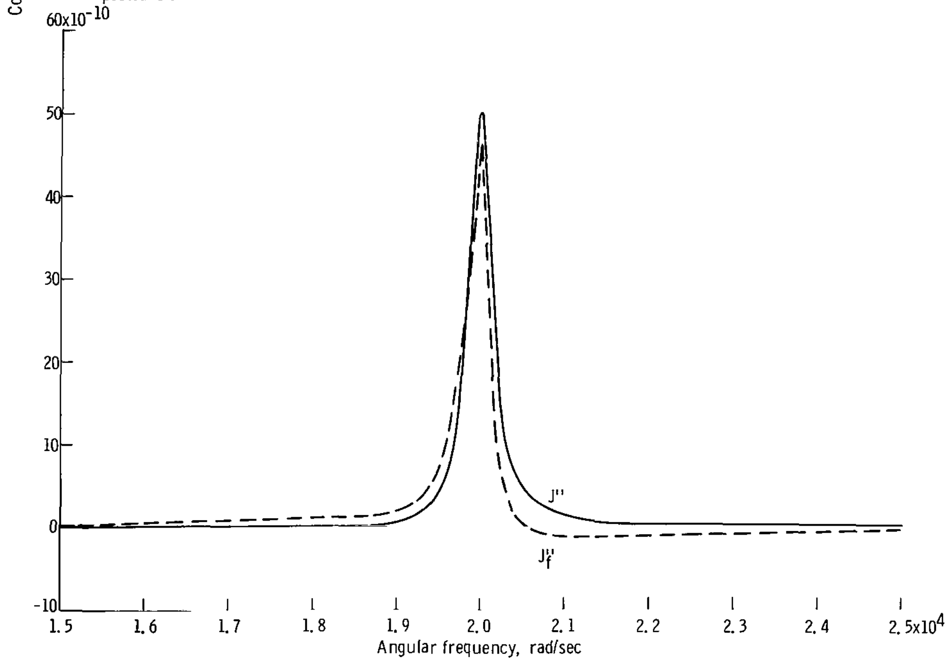
Substituting Z_m^R from equation (26) for Z'_m in equation (25), the effect of the shorted turn drive tube on Z_m^R is

$$Z'_{mf} = \left[\rho - \frac{\omega L_c}{R_c}\left(\omega m - \frac{k}{\omega}\right)\right] + i\left(\omega m - \frac{k}{\omega} + \frac{\omega L_c \rho}{R_c}\right) \tag{27}$$

The shorted turn drive tube thus causes a mixing of the real and imaginary parts of the mechanical impedance.



(a) Storage compliances J' and J'_f calculated for nonshorted and shorted drive tube, respectively. Mixing of storage J' and loss J'' compliances causes shorted tube storage compliance J'_f to depart from expected J'.



(b) Loss compliance J'' and J''_f calculated for nonshorted and shorted drive tube, respectively. Particularly note way in which mixing of J' with J'' caused by shorted turn drive tube causes J''_f to become negative just above 2.05 radians per second. This negative part accounts for unphysical negative portion of loss compliance first reported by Fitzgerald.

Figure 4. - Comparison of calculated vector complex compliances with and without shorted turn effects. Curves were calculated by using equations (27) and (30) with constants: $m = 31.4$ grams; $k = 1.256\times10^{10}$ dynes per centimeter; $L_c/R_c = 1.49\times10^{-5}$ henry per ohm; $\rho = 10^4$ dyne second per centimeter.

Fitzgerald defines a complex compliance J^* as

$$J^* = -\frac{ic}{\omega Z_m'} \tag{28}$$

where c is a constant which depends on the dimensions of the sample. Letting

$$J^* = J' - iJ'' \tag{29}$$

the effect of the shorted turn drive tube on the storage and loss compliance (J' and J'' respectively) is;

$$\begin{aligned} J_f' &= \left[\frac{1}{1 + \left(\frac{\omega L_c}{R_c}\right)^2}\right]\left(J' - \frac{\omega L_c}{R_c} J''\right) \\ J_f'' &= \left[\frac{1}{1 + \left(\frac{\omega L_c}{R_c}\right)^2}\right]\left(J'' + \frac{\omega L_c}{R_c} J'\right) \end{aligned} \tag{30}$$

From these two equations it can be seen that the shorted turn drive tube causes a mixing of the real and imaginary parts of the complex compliance. In figure 4 is shown a comparison of calculated compliances, with and without shorted turn effects, for a resonance near 3000 hertz. The constants were chosen to give a rather typical resonance as judged by comparison with the resonances observed by Fitzgerald. The important point to observe is that the mixing of J' with J'' to form J_f'' causes this part of the compliance to become negative just above the resonance in a way which closely resembles the negative loss reported by Fitzgerald.

CONCLUSIONS

In the previous section it was shown that the aluminum portion of the drive tube located under the driving coil acts as a single turn secondary of a transformer and a shorted turn moving in a magnetic field, which results in a frequency dependent mixing of the real and imaginary parts of the sample impedance and compliance. Because the magnitudes of

the anomalous terms depend linearly on frequency, resonances which occur at high frequencies should show a larger mixing than those which occur at low frequencies. This effect is observed in Fitzgerald's experimental results where for resonances below about 1.5 kilohertz the loss compliance appears to remain positive, while above this frequency the tendency for the loss compliance to become negative increases with increasing frequency as evidenced by the increasing asymmetry of J'' about the resonant frequency. For many resonances near 3000 hertz and above the loss compliance is negative.

The magnetic damping, produced by the eddy currents induced in the shorted turn of the drive tube is usually much larger than the damping produced by most samples. As a result a precise measurement of sample damping is impossible since the sample damping is determined by a subtraction of damping terms which contain a large magnetic contribution.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, November 20, 1968,
129-03-15-01-22.

APPENDIX - SYMBOLS

$\vec{B}$ magnetic induction vector

$\vec{B}_c$ magnetic induction at shorted turn

$\vec{B}_1$ magnetic induction in coil 1A gap

$\vec{B}_2$ magnetic induction in coil 2A gap

B^o_{12} imaginary term in mechanical impedance of free drive tube

B^o_{12f} combination of shorted turn effects and B^o_{12}

C_B phase shift capacitance in Fitzgerald's circuit diagram

C_1 capacitance in Fitzgerald's bridge

C_4 capacitance in Fitzgerald's bridge

c Fitzgerald's sample geometry constant

D_c line integral of B_1 around shorted turn

D_1 line integral of B_1 around coil 1A

D_2 line integral of B_2 around coil 2A

$\vec{dl}$ increment of length coil 1A

$\vec{dl}_c$ increment of length; shorted turn

dl_2 increment of length; coil 2A

E voltage applied to Fitzgerald's bridge

E_a voltage induced in either coil due to motion in magnetic field

$E^{\pm}_{2A}$ voltage across coil 2A; superscript denotes connection to bridge

$\vec{e}_v$ unit vector along v

$F^{\pm}$ force on drive tube

G^o_{12} real term in mechanical impedance of free tube

G^o_{12f} mixture of shorted turn effects and G^o_{12}

$I^{\pm}_c$ current in shorted turn; $\pm$ denotes connection of 2A

I_1 current in coil 1A

I_2 current in coil 2A

$I^{\pm}_2$ current in coil 2A; connection of 2A in bridge explicit

J' real term of compliance J^*

J'' imaginary term of compliance J^*

J^* vector complex compliance

K^2 product of $D_1 \cdot D_2$

L_c inductance of shorted turn

M_{1AC} mutual inductance between shorted turn and coil 1A

M^o mass of free tube

n number of turns in coil 1A

R_A resistor in Fitzgerald's bridge

R_B resistor in Fitzgerald's bridge

R_c resistance of the shorted turn

R_1 resistor in Fitzgerald's bridge

R_3 resistor in Fitzgerald's bridge

R_4 resistor in Fitzgerald's bridge

S^o stiffness of drive tube suspension

$\vec{v}$ vector velocity

$v^{\pm}$ velocity of drive tube; $\pm$ denotes connection of coil 2A

ΔZ difference of electrical impedances

Z_c electrical impedance of shorted turn

Z_m mechanical impedance

Z'_m mechanical impedance of sample

Z_m^c mechanical impedance with drive tube clamped to the floating mass

Z_m^o mechanical impedance of free drive tube

Z_m^R mechanical impedance of oscillator

Z_{mf} mixture of true mechanical impedance and shorted turn effects

Z'_{mf} apparent mechanical impedance of sample

Z_{mf}^o mixture of Z_m^o and shorted turn effects

Z_{1AC} electrical impedance defined as $-i\omega M_{1AC}$

$(Z_2)^{\pm}$ electrical impedance of coil 2A; connection of 2A denoted by $\pm$; includes mechanical effects

Z_{2A} electrical impedance of coil 2A, ohmic

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